A STUDY OF NATURAL CONVECTION IN HORIZONTAL CYLINDRICAL GAPS

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A study was made of natural convection in horizontal cylindrical gaps. For hot tubes of small diameters, the convection factor has been related not only to the Rayleigh number but also to the geometrical dimensions (diameter of the hot tube and relative gap width). The construction of experimental tubes is also described.

In earlier reports [1, 2] the author has presented the results of an experimental study concerning the natural convection of heat from thin wires into the surrounding enclosed space. The author analyzed there how the convection factor depends on the gap width δ, on the wire length, and on the spatial orientation of the wire.

In this study the author analyzed the effect of the diameter of the hot cylinder on the rate of natural convection during heat transmission through horizontal cylindrical-annular gaps. The heat was supplied from an inner hot cylinder through a fluid (distilled water or 96% ethanol) to an outer cylinder. The convection factor

$$\varepsilon = \frac{\lambda_{\text{eff}}}{\lambda} \tag{1}$$

in this experiment was determined for tubes of equal lengths and with equal gap widths but different diameters of the inner and the outer cylinders.

If the amount of transmitted heat and the temperature drop between both cylinders are known, then the effective thermal conductivity

$$\lambda_{\text{eff}} = \frac{AQ}{\Delta T}, \tag{2}$$

with $A = \dfrac{\ln \dfrac{D_a}{D_i}}{2\pi l}$ can be determined by integrating the Fourier equation. The effective thermal conductivity λ_{eff} in tubes Nos. 1, 2, and 3 was measured by the "hot wire" method. A description of this method can be found in [1, 3]. The dimensions of those tubes are given in Table 1.

Tube No. 1 was designed as conventionally for this method of measurement. It was made of molybdenum-grade glass with a platinum wire stretched along the axis and heated with direct electric current. On the outside surface of the tube was wound another platinum wire serving as a resistance thermometer. In tubes Nos. 2 and 3 the platinum wire along the axis was replaced by a thin-wall nickel tubule also heated with direct electric current. This nickel tubule was centered inside each glass tube by means of two Teflon washers. These washers, with the tubule in between, also fixed the length of the test zone. Each tube was then plugged on both ends. The temperature of the hot tube and that of the cold tube were determined from the resistance of the nickel tubule and that of the external platinum thermometer. The amount of transmitted heat was determined from the current and the voltage drop across the test segment.

The experimental tube No. 4 was designed differently. The hot cylinder was made of stainless steel. Inside it were placed three heater elements in the form of wire coils wound around glass tubes; the main heater along the test segment and the two auxiliary ones at the cylinder ends. The temperature of the hot cylinder was measured with six copper–constantan thermocouples fused into the cylinder wall. A brass

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TABLE 1. Dimensions of the Tubes

Item No.	l, mm	D_i, mm	D_a, mm	δ, mm	$\frac{D_a}{D_i}$	$\frac{\delta}{D_i}$
1	130	0,1	13,3	6,6	133	66
2	130	1	12,85	5,92	12,85	5,92
3	130	3,35	16,51	6,58	4,95	1,96
4	130	14,1	27,67	6.785	1,96	0,48

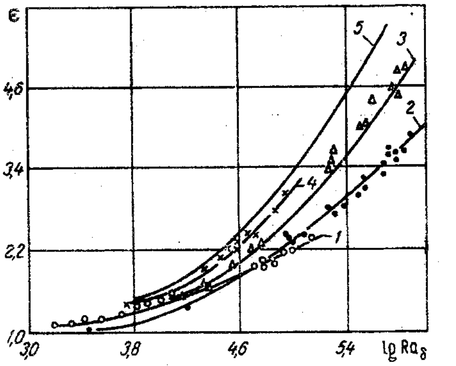

Fig. 1

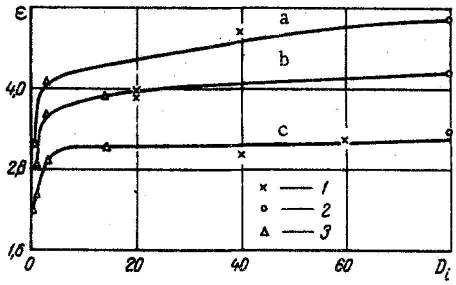

Fig. 2

Fig. 1. Convection factor ε as a function of the Rayleigh number Ra_δ (plotted to a semilog scale).

Fig. 2. Convection factor $\varepsilon = f(D_i)_{Ra_\delta} = \text{const}$ as a function of the diameter of the hot cylinder D_i (mm): 1) test values from [4]; 2) test values from [5]; 3) test values from this study; a) $Ra_\delta = 3.2 \cdot 10^5$; b) $Ra_\delta = 2 \cdot 10^5$; c) $Ra_\delta = 1 \cdot 10^5$.

tube served as the outer cylinder. Its temperature was also measured with six copper–constantan thermocouples. The temperatures were measured by the compensation method. The entire assembly was placed inside a fluid thermostat.

The test data were evaluated in terms of the criterial relation

$$\varepsilon = f(Ra_\delta). \tag{3}$$

Here $Ra_\delta = Gr_\delta Pr$, $Gr_\delta = \beta\rho^2 g\Delta T\delta^3/\eta^2$; $Pr = \eta c_p g/\lambda$.

The thickness δ of the fluid layer was selected as the reference dimension for calculating the Rayleigh number. The results are shown in Fig. 1. Curves 1, 2, 3, and 4 refer to tubes Nos. 1, 2, 3, and 4, respectively. Curve 5 represents the values of ε based on the Crousehold–Mikheev equation

$$\varepsilon = 0.105\,(Ra_\delta)^{0.3}. \tag{4}$$

The graph indicates that, for coaxial cylinders of equal lengths l, and of equal gap widths δ, the convection factor depends not only on the Rayleigh number but also on the diameter of the hot inner cylinder D_i: it decreases when this diameter becomes smaller. The Crousehold–Mikheev equation yields values of the convection factor which are too high for small cylinders.

In order to determine the effect of D_i, should that diameter of the inner cylinder be varied over a range as a wide as in this study, Grigul and Hauf [4] as well as Beckmann [5] had plotted curves of $\varepsilon = f(D_i)$ for constant values of Ra_δ (Fig. 2). It is quite evident that the dependence of the convection factor on the diameter of the hot cylinder becomes gradually weaker and eventually ceases as this diameter becomes larger. An explanation for such a trend is, apparently, that at small diameters D_i the development of a free flow around a hot horizontal cylinder depends largely on the size of the surface ($2\pi r_i$) along which the heated liquid flows.

Let us now consider the convection factor as a function of the relative gap width. The curves of $\varepsilon = f(\delta/D_i)$ in Fig. 3 have been plotted for constant values of Ra_δ on the basis of the test data in this study as well as in [4] and [5]. It is quite evident that the convection factor becomes maximum when $\delta/D_i \simeq 1$. It decreases, as the relative gap width is further increased up to $\delta/D_i \simeq 6$ and levels off to an almost constant value when $\delta/D_i > 6$. The trend of this relation should be of some significance for the solution of engineering problems.

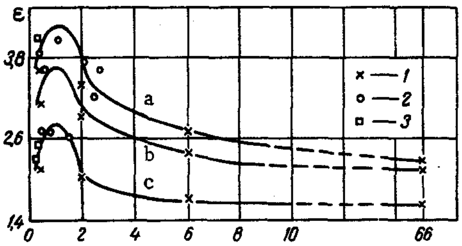

Fig. 3

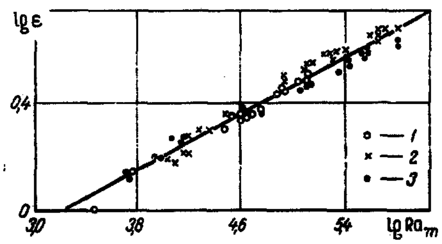

Fig. 4

Fig. 3. Convection factor as a function of the relative gap width, at constant values of Ra_δ: 1) according to this study; 2) according to the data in [4]; 3) according to the data in [5]; a) $Ra_\delta = 3.2 \cdot 10^5$; b) $2 \cdot 10^5$; c) $1 \cdot 10^5$.

Fig. 4. Convection factor ε as a function of Ra_m (plotted to a log–log scale), based on measurements made in: 1) tube No. 4 (D_i = 14.1 mm); 2) tube No. 3 (D_i = 3.35 mm); 3) tube No. 2 (D_i = 1.0 mm).

Several empirical equations for calculating the convection factor have been proposed in [1, 4 , 5]. The test data of this study, except those pertaining to tube No. 1 (D_i = 0.1 mm), may be generalized by the critical equation

$$\varepsilon = 0.17\,(Ra_m)^{0.25}, \tag{5}$$

applicable within the range $1600 < Ra_m < 600{,}000$.

Here, as has been suggested in [6], the characteristic length $m = \sqrt{r_a r_i}\ln r_a/r_i$ is used for calculating the Rayleigh number and, consequently,

$$Ra_m = \frac{\beta\rho^2 g\Delta T}{\eta^2}\left[\sqrt{r_a r_i}\,\ln\frac{r_a}{r_i}\right]^3 Pr.$$

The results of these calculations are shown in Fig. 4. The scatter of test values does not exceed 10% and is random, related to the measurement error as well as to the error in the values of the thermophysical properties used for calculating the Rayleigh number.

It is to be noted that the deviation of test points from curve 5 in Fig. 1, which has been calculated according to the Crousehold–Mikheev equation, is systematic and dependent on the diameter D_i. For small diameters D_i this deviation of test points from the universal curve is up to almost 50%.

NOTATION

l	is the length of the test zone;
r_i, D_i	are the outside radius and diameter, respectively, of the hot inner cylinder;
r_a, D_a	are the inside radius and diameter, respectively, of the outside cylinder;
δ	is the gap width;
λ_{eff}	is the effective thermal conductivity;
A	is the instrument constant;
Q	is the amount of transmitted heat;
ΔT	is the temperature drop between hot and cold cylinder;
ε	is the convection factor;
λ	is the molecular thermal conductivity;
Gr_δ	is the Grashof number referred to the gap width;
Pr	is the Prandtl number;
β	is the thermal volume expansivity;
ρ	is the density;
g	is the acceleration due to gravity;
η	is the dynamic viscosity;
c_p	is the specific heat at constant pressure;
Ra_δ	is the Rayleigh number referred to the gap width;
m	is the characteristic dimension;
Ra_m	is the Rayleigh number referred to the characteristic dimension.

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